2026 **ANDRE TUTORING SAMPLE EXAM**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9–36)

- Attempt Questions 11–29
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 For what real values of k will the quadratic inequality $x^2 - 2(k-1)x + (k+5) > 0$ hold true for all real values of x?

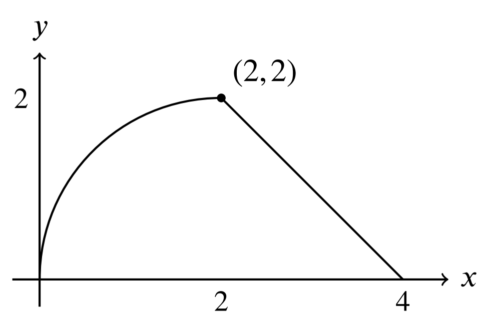
A. $-1 < k < 4$

B. $k < -1$ or $k > 4$

C. $-2 < k < 4$

D. $k < -2$ or $k > 4$

2 The graph below tracks a continuous function $y = f(x)$ defined on the domain $0 \le x \le 4$. The curve consists of a quarter-circle of radius 2 centered at $(2,0)$, connected to a straight line.



Find the exact value of the definite integral $\int_0^4 f(x)\,dx$.

A. $\pi - 2$

B. $\pi + 2$

C. $2\pi - 2$

D. $2\pi + 2$

3 Consider a geometric sequence where the sum of the first two terms is 15, and the sum of the next two terms is 60. Given that the common ratio r is negative, determine the exact value of the first term a.

A. -15

B. -5

C. 5

D. 15

4 Consider the quadratic function $f(x) = x^2 - 3x$. Which of the following expressions represents the equation of the tangent line to the curve at the point where $x = p$?

A. $y = (2p-3)x - p^2$

B. $y = (2p-3)x + p^2$

C. $y = (2p-3)x - p^2 + 3p$

D. $y = 2px - p^2 - 3$

5 Let A, B, and C be three events such that $P(A) = P(B) = P(C) = \frac{1}{3}$. It is known that the events are pairwise independent, meaning $P(A \cap B) = P(A)P(B)$, $P(B \cap C) = P(B)P(C)$, and $P(A \cap C) = P(A)P(C)$.

If the probability of the mutual intersection of all three events is zero, determine the exact value of the probability of the union of these three events, $P(A \cup B \cup C)$.

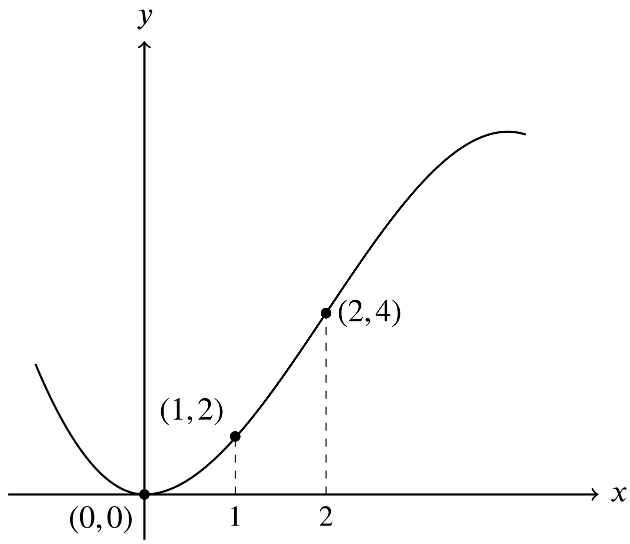
A. $\frac{1}{3}$

B. $\frac{4}{9}$

C. $\frac{2}{3}$

D. 1

6 The graph of a continuous, twice-differentiable function $y = f(x)$ is shown below for the domain $-1.5 \leq x \leq 4.5$. The curve has a sharp local minimum turning point at $(0,0)$, an inflection point at $(1,2)$, and a local maximum turning point at $(3,4)$.



Which of the following inequality sets correctly describes the values of the function and its respective derivatives evaluated precisely at the coordinate position $x = 1$?

A. $f''(1) < f'(1) < f(1)$

B. $f'(1) < f''(1) < f(1)$

C. $f''(1) < f(1) < f'(1)$

D. $f(1) < f'(1) < f''(1)$

7 Let $f(x)$ be a differentiable function defined for all real numbers such that $f(1) = 2$ and $f(4) = 8$. Evaluate the exact value of the following definite integral:

$$\int_1^2 \frac{x f'(x^2)}{f(x^2)}\, dx$$

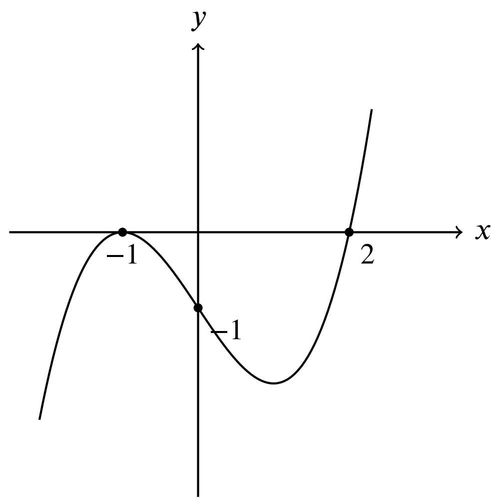
A. $\ln 8$

B. $\ln 4$

C. $\frac{1}{2}\ln 2$

D. $\ln 2$

8 The graph of a polynomial function $y = P(x)$ is shown below:



Which of the following equations best matches the graph displayed?

A. $y = \frac{1}{2}(x-1)^2(x+2)$

B. $y = \frac{1}{2}(x+1)(x-2)^2$

C. $y = \frac{1}{2}(x+1)^2(x-2)$

D. $y = (x+1)^2(x-2)$

9 The population P of a bacterial culture at time t hours is modeled by the function:

$$P(t) = 500 + 200 \cdot t \cdot e^{-\frac{t}{4}}$$

Determine the exact rate of change of the population with respect to time at the instant when $t = 4$ hours.

A. 0

B. $\frac{200}{e}$

C. $-\frac{200}{e}$

D. $500 + \frac{800}{e}$

10 Let X be a continuous random variable that follows a perfect normal distribution centered at a mean of μ. If it is known that $P(X > \mu + 2) = p$, which of the following expressions represents the exact value of the conditional probability $P(X < \mu + 2 \mid X > \mu - 2)$?

A. $\dfrac{1-p}{1+p}$

B. $\dfrac{1-2p}{1-p}$

C. $\dfrac{p}{1-p}$

D. $\dfrac{2p}{1-p}$

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Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–29
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (6 marks)

A particle moves in a straight line so that its velocity v metres per second at time t seconds ($t \geq 0$) is given by:

$$v = \frac{40t}{(2t^2+5)^2}$$

Initially, the particle is located 3 metres to the right of a fixed origin O.

(a) Find an expression for the position x of the particle relative to O at any time t. **2**

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(b) Determine the maximum velocity achieved by the particle. **3**

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(c) Describe the limiting behaviour of the position of the particle as $t \to \infty$. **1**

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End of Question 11

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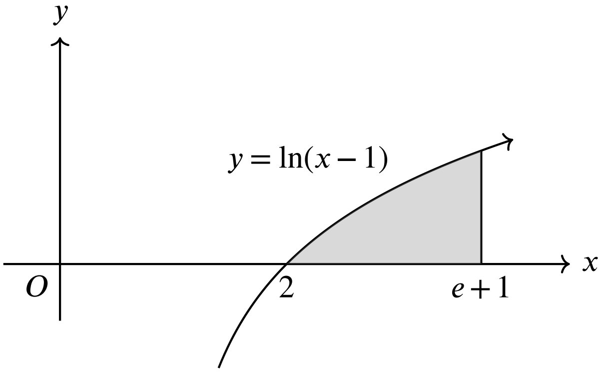
Question 12 (4 marks)

The diagram below shows the region bounded by the curve $y = \ln(x-1)$, the x-axis, and the vertical line $x = e+1$.



(a) By integrating with respect to the y-axis, or otherwise, calculate the exact area of the shaded region. **3**

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(b) A student uses the trapezoidal rule with three function values to approximate the area of the same shaded region. **1**

Without performing any calculations, state whether this approximation will be an underestimate or an overestimate of the true area.

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End of Question 12

Question 13 (5 marks)

A continuous random variable X has a probability density function given by:

$$f(x) = \begin{cases} ke^{-2x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

Where k is a positive constant.

(a) Show that $k = 2$. **1**

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(b) Find the exact value of the median of X. **2**

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(c) Calculate the probability $P(X \leq 1 \mid X \geq 0.5)$, giving your answer correct to three decimal places. **2**

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End of Question 13

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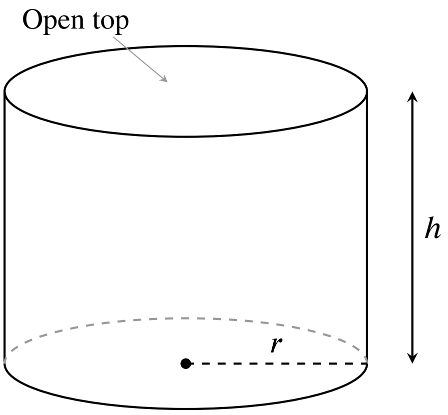
Question 14 (6 marks)

An open cylindrical waste bin of radius r cm and height h cm is manufactured such that the base and the curved vertical wall have an area of exactly 1200π cm^2.



(a) Show that the volume V cm^3 of the waste bin is given by $V = 600\pi r - \frac{1}{2}\pi r^3$. **2**

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(b) Find the dimensions r and h that will maximize the volume of the bin. Justify that your solution yields a maximum volume. **4**

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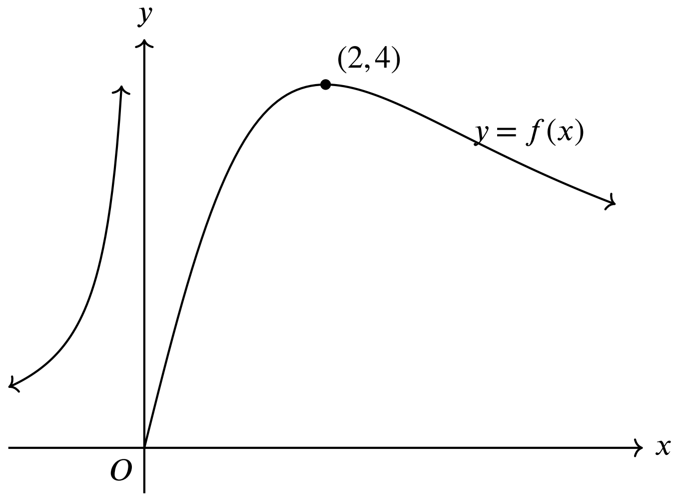
End of Question 14

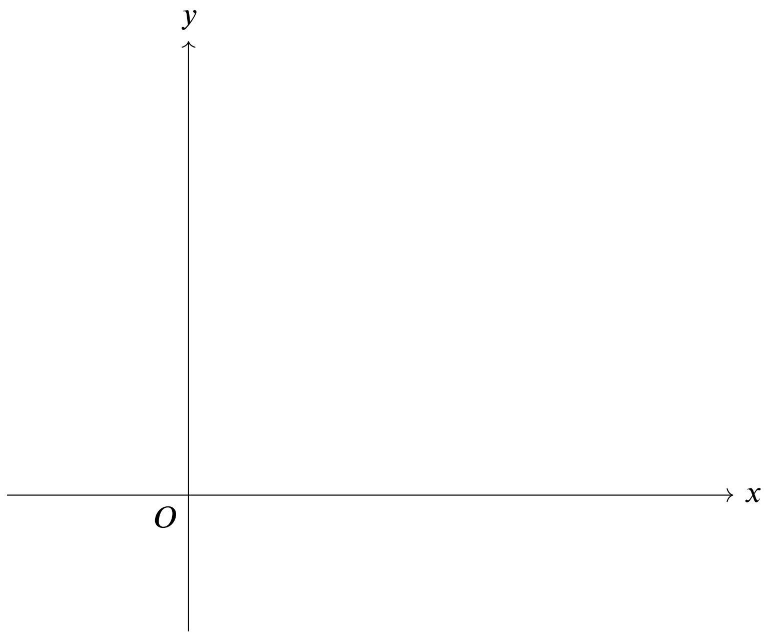
Question 15 (5 marks)

The diagram below shows the graph of a function $y = f(x)$. The graph has a horizontal asymptote at $y = 0$, a vertical asymptote at $x = 0$, and a local maximum turning point at $(2, 4)$.



(a) On the axes provided below, sketch the graph of $y = 2f(x) - 1$. **3**

Clearly label the coordinates of the new turning point and the equations of any new asymptotes.



Question 15 continues on next page

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(b) State the domain and range of the transformed function $y = 2f(x) - 1$, given that $f(x) > 0$ for all x, has one turning point and is continuous for all values other than $x = 0$. **2**

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Question 16 (5 marks)

A company manufactures designer water bottles. The marginal revenue, R, in dollars per bottle, obtained by selling x bottles is given by the derivative function:

$$\frac{dR}{dx} = 150 - 0.4x$$

The marginal cost, C, in dollars per bottle, to manufacture x bottles is given by:

$$\frac{dC}{dx} = 0.2x + 30$$

The company's fixed costs before any bottles are produced are \$5000.

(a) Find the number of bottles, x, that must be produced and sold each week to maximize total profit. **2**

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(b) Determine the maximum weekly profit achievable by the company. **3**

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Question 17 (6 marks)

An financial analyst models the balance of an investment portfolio using a geometric sequence. A client invests \$2000 at the start of every year into an account that pays a fixed compound interest rate of 4% per annum, with interest paid at the end of each year.

Let A_n be the total value of the investment at the end of the n-th year, immediately after interest has been paid.

(a) Show that the total value of the investment at the end of 3 years is given by: **2**

$$A_3 = 2000(1.04) + 2000(1.04)^2 + 2000(1.04)^3$$

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(b) Hence, derive an expression for A_n and calculate the total amount in the investment account at the end of 25 years, rounding your final answer to the nearest dollar. **4**

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End of Question 17

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Question 18 (4 marks)

Consider the function given by $f(x) = x\sqrt{4 - x^2}$ for $-2 \leq x \leq 2$.

(a) Show that the derivative of $f(x)$ is given by $f'(x) = \dfrac{4 - 2x^2}{\sqrt{4 - x^2}}$. **2**

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(b) Hence, find the exact value of $\displaystyle\int_0^1 \frac{x^2 - 2x - 2}{\sqrt{4 - x^2}}\,dx$. **2**

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Question 19 (4 marks)

A high-tech manufacturing company produces smartphone screens. The thickness of these screens is normally distributed with a mean μ of 0.400 mm and a standard deviation σ of 0.015 mm.

Any screen with a thickness less than 0.370 mm or greater than 0.415 mm is considered defective and cannot be sold.

(a) Using the empirical rule, calculate the percentage of manufactured screens that are expected to be defective. **2**

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(b) The engineering team introduces a new calibration method that keeps the mean thickness at 0.400 mm but decreases the standard deviation to a new value, σ. **2**

If the company requires that exactly 97.5% of all produced screens must now meet the criteria to be non-defective, find the value of σ assuming the upper threshold of 0.425 mm determines this boundary.

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Question 20 (5 marks)

An infinite geometric series is given by:

$$1+\frac{1}{2}\sec\left(2x-\frac{\pi}{6}\right)+\frac{1}{4}\sec^2\left(2x-\frac{\pi}{6}\right)+\frac{1}{8}\sec^3\left(2x-\frac{\pi}{6}\right)+\ldots$$

Where x is restricted to the interval $0 \leq x \leq \pi$.

(a) Find all values of x within the interval $0 \leq x \leq \pi$ for which this geometric series has a limiting sum. **3**

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(b) Hence, determine the exact value of x in the interval $0 \leq x \leq \frac{\pi}{2}$ such that the limiting sum of the series is exactly equal to $\dfrac{4}{4-\sqrt{2}}$. **2**

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End of Question 20

Question 21 (7 marks)

Biologists monitor the population dynamics between a native bird species and an invasive cicada population on an isolated island.

The bird population, $B(t)$, and the cicada population, $C(t)$ (measured in thousands), are modelled over a continuous time period by the following trigonometric functions:

$$B(t) = 500 + 150\sin\left(\frac{\pi t}{6}\right)$$
$$C(t) = 80 + 40\sin\left(\frac{\pi t}{6} - \frac{\pi}{4}\right)$$

where t is the time in months ($0 \le t \le 12$), with $t = 0$ representing the beginning of January.

(a) Determine the time lag, in months, between the cicada population reaching its absolute maximum and the bird population reaching its absolute maximum. **2**

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(b) Calculate the exact rate of change of the cicada population at the precise moment when the bird population is decreasing at its absolute maximum rate. Include appropriate units. **3**

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(c) On the single set of axes, sketch the graphs of both $y = B(t)$ and $y = C(t)$ for the interval $0 \le t \le 12$. **2**

Question 22 (2 marks)

Find all exact solutions to the equation: **2**

$$\tan 2\theta + \cot 2\theta = 4$$

For the interval $0 \le \theta \le \pi$.

Question 23 (5 marks)

Consider the function $f(x) = \dfrac{\ln x}{x^2}$ defined for $x > 0$.

(a) Find the exact coordinates of the stationary point of $y = f(x)$ and determine its nature. **3**

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(b) Find the coordinates of the point of inflection of the curve. **2**

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End of Question 23

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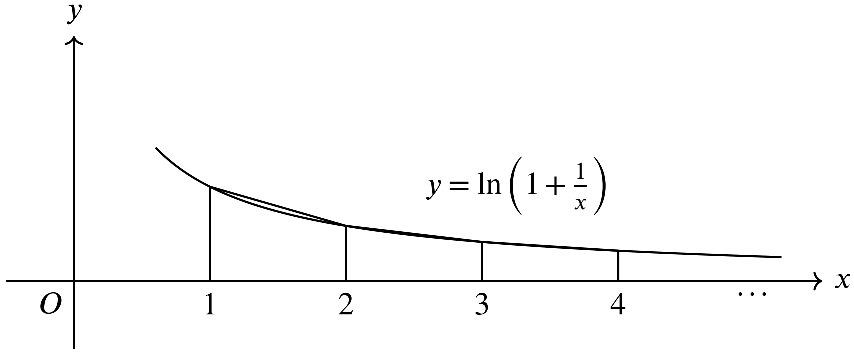
Question 24 (6 marks)

An environmental scientist wants to approximate the area under the curve $y = \ln\left(1+\frac{1}{x}\right)$ from $x = 1$ to $x = n+1$, where n is a positive integer.

Instead of using regular intervals, the scientist divides the domain into n unequal subintervals where the vertical grid lines are placed at successive integer values $x = 1,\ 2,\ 3,\ \ldots,\ n,\ n+1$, forming a series of trapeziums.



(a) Show that the area of the single trapezium formed between $x = k$ and $x = k+1$ is given by: **1**

$$A_k = \frac{1}{2}\left[\ln\left(\frac{k+1}{k}\right) + \ln\left(\frac{k+2}{k+1}\right)\right]$$

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(b) By summing the areas of all n trapeziums, show that the total approximated area simplifies to: **3**

$$A_{\text{total}} = \frac{1}{2}\left[\ln\left(\frac{n+2}{2}\right) + \ln(n+1)\right]$$

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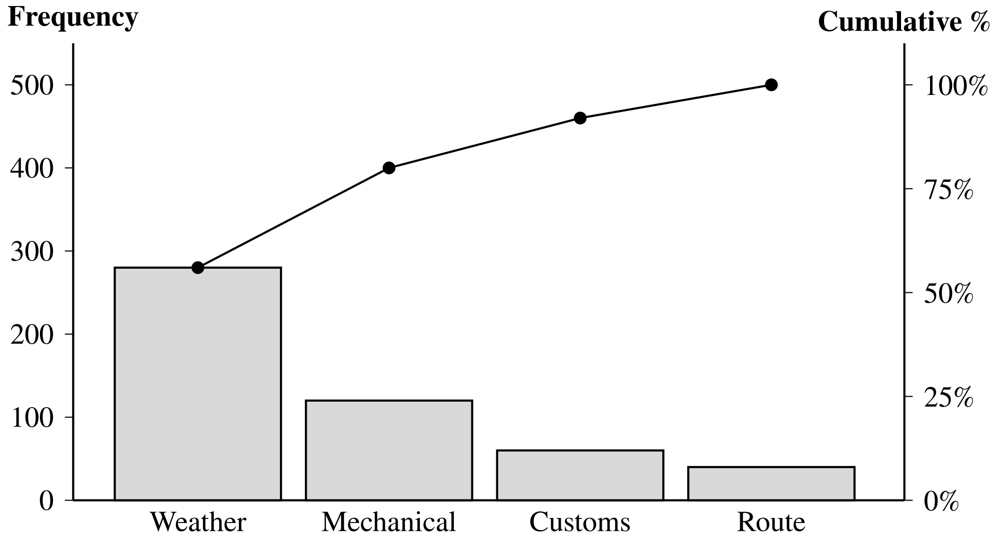
(c) Find the value of the approximated area when $n = 62$, expressing your answer in the form $\ln(p)$. **2**

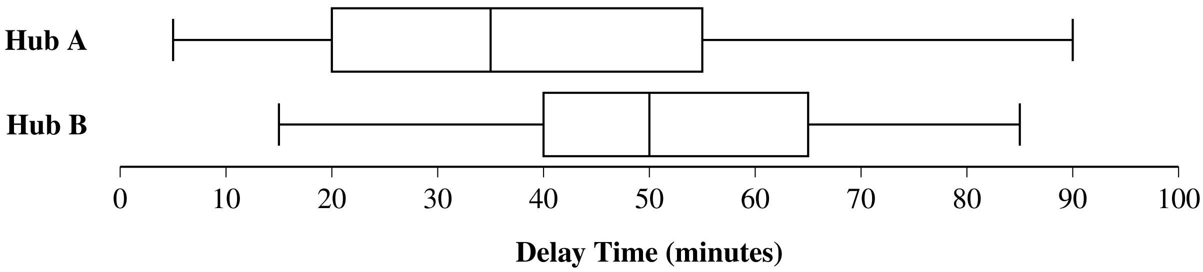
End of Question 24

Question 25 (5 marks)

A logistics firm analyses transit delays across two transport hubs, Hub A and Hub B. The Pareto chart below highlights the frequency and cumulative percentage of specific categories of delays recorded at Hub A over a three-month observation window.



In addition, the parallel box-and-whisker plots below illustrate the overall transit delay times (in minutes) recorded for shipments passing through Hub A and Hub B.



(a) Using the Pareto chart, identify the categories of delays that constitute the standard 80% threshold of transit problems at Hub A. **1**

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(b) Calculate and compare the delay times between Hub A and Hub B. What does this reveal about the consistency of delays at both hubs? **2**

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(c) An analyst reviews the raw data from the Pareto chart and notes that Hub A recorded a total of 500 delays. The data reveals that: **2**

- 25% of *Mechanical* delays exceed 55 minutes
- 5% of *Customs* and *Route* delays combined exceed 55 minutes.
- However, the record for the percentage of *Weather* delays exceeding 55 minutes is missing.

Hence, approximate the percentage of *Weather* delays that must have exceeded 55 minutes.

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End of Question 25

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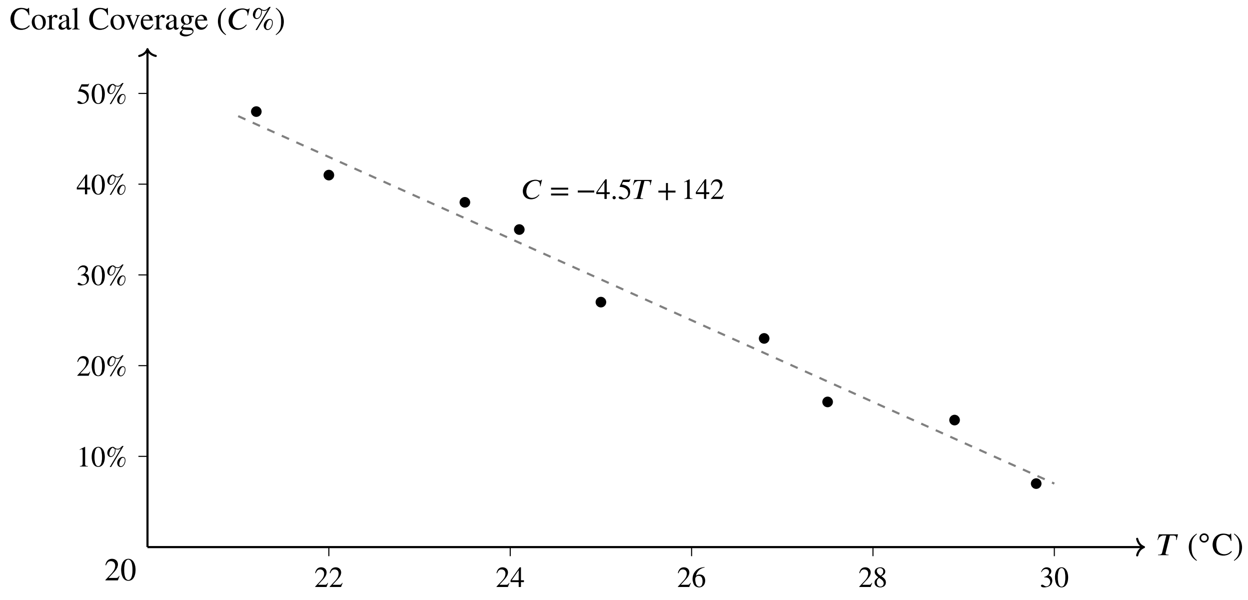
Question 26 (3 marks)

A marine biologist monitors the coral coverage percentage (C) at a local reef as a function of the average surface water temperature (T, in °C).

Over several months of data tracking, the biologist records the scatter plot below and computes a linear line of best fit. The Pearson's correlation coefficient for this dataset is calculated to be $r = -0.87$.



(a) Interpret the meaning of both the gradient and the Pearson's correlation coefficient in the context of this biological scenario. **2**

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(b) At a monitored temperature of 24°C, the actual observed coral coverage is found to be 35%. Calculate the exact difference between this observed value and the value predicted by the linear model. **1**

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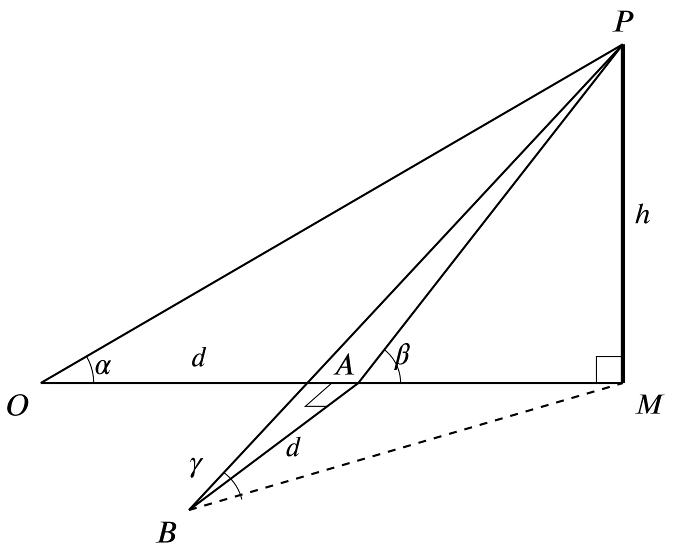
Question 27 (4 marks)

A surveyor at O on a flat plain observes a vertical peak P at an angle of elevation α. The surveyor walks d metres directly towards the mountain base to A, where the angle of elevation is β. From A, the surveyor turns 90° left and walks d metres to B. At B, the angle of elevation to P is γ.



(a) Let h represent the vertical height of the peak P above its base M. Show that the horizontal distance from A to M is $h\cot\beta$. **1**

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(b) Hence, prove that: **3**

$$\cot^2\gamma = \cot^2\beta + (\cot\alpha - \cot\beta)^2$$

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Question 28 (3 marks)

A discrete random variable X has the probability distribution shown below, where $E(X)$ represents the expected value of the distribution: **3**

x	1	2	3	4	5
$P(X = x)$	$\dfrac{a}{E(X)}$	$\dfrac{a}{E(X)}$	$\dfrac{a}{E(X)}$	$\dfrac{2a}{E(X)}$	$\dfrac{a}{2E(X)}$

Determine the exact value of the constant parameter a.

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End of Question 28

Question 29 (5 marks)

Consider the function $f(x) = \dfrac{x^2 \cos x}{1+e^x}$ defined over the domain $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$.

It is known that $\displaystyle\int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} f(-x)\,dx$.

Let I represent the definite integral:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x}\,dx$$

(a) Hence, show that $I = \dfrac{1}{2}\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\,dx$ **2**

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(b) By considering the graph of $f(x) = x^2 \cos x$, over the interval $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$. **3**

Justify why the calculated area I must be strictly less than $\dfrac{\pi^3}{24}$.

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Mathematics Advanced Paper
Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	B
3	A
4	A
5	C
6	A
7	D
8	C
9	A
10	B

Section II

Question 11 (a)

Criteria	Marks
• Correctly integrates the velocity expression and solves for the constant	2
• Finds the correct primitive integration form, or equivalent merit	1

Sample answer:

$$x(t) = \int \frac{40t}{(2t^2+5)^2}\,dt$$

$$x(t) = 10\int \frac{4t}{(2t^2+5)^2}\,dt$$

$$= -\frac{10}{2t^2+5} + C$$

$$3 = -\frac{10}{2(0)^2+5} + C$$

$$3 = -2 + C$$

$$C = 5$$

$$x(t) = 5 - \frac{10}{2t^2+5}$$

Question 11 (b)

Criteria	Marks
• Determines the correct exact maximum velocity value	3
• Solves for stationary point condition time t value	2
• Differentiates $v(t)$ correctly using calculus quotient rules	1

Sample answer:

$$\frac{dv}{dt} = \frac{40(2t^2+5)^2 - 40t\cdot 2(2t^2+5)(4t)}{(2t^2+5)^4}$$

$$= \frac{40(2t^2+5) - 320t^2}{(2t^2+5)^3}$$

$$= \frac{200 - 240t^2}{(2t^2+5)^3}$$

$$0 = \frac{200 - 240t^2}{(2t^2+5)^3}$$

$$0 = 200 - 240t^2$$

$$t^2 = \frac{5}{6}$$

$$t = \sqrt{\frac{5}{6}} \text{ seconds}$$

$$v_{\text{max}} = \frac{40\sqrt{\frac{5}{6}}}{\left(2\left(\frac{5}{6}\right)+5\right)^2}$$

$$= \frac{40\sqrt{\frac{5}{6}}}{\left(\frac{20}{3}\right)^2}$$

$$= \frac{9\sqrt{30}}{20} \text{ m/s}$$

Question 11 (c)

Criteria	Marks
• Correctly describes the limiting asymptotic horizontal position	1

Sample answer:

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty}\left(5 - \frac{10}{2t^2+5}\right)$$

$$= 5 - 0$$

$$= 5$$

$\therefore$ The particle approaches a position 5 metres to the right of origin O.

Question 12 (a)

Criteria	Marks
• Provides the correct exact area value	3
• Evaluates boundaries correctly and forms the inverse variable function	2
• Formulates the correct template layout for bounding integration	1

Sample answer:

$$y = \ln(x-1)$$

$$e^y = x - 1$$

$$x = e^y + 1$$

$$A = \int_0^1 \left[(e+1) - (e^y+1)\right] dy$$

$$= \int_0^1 (e - e^y)\, dy$$

$$= \left[ey - e^y\right]_0^1$$

$$= (e-e) - (0-1)$$

$$A = 1$$

Question 12 (b)

Criteria	Marks
• States correct approximation nature with sound geometric curvature justifications	1

Sample answer:

Because the curve $y = \ln(x-1)$ is strictly concave down $\left(\dfrac{d^2y}{dx^2} < 0\right)$, any secant line connecting two points on the curve lies below the true graph, meaning the trapeziums will underestimate the area under the curve.

Question 13 (a)

Criteria	Marks
• Successfully verifies normalization total area criteria conditions	1

Sample answer:

$$\int_0^\infty ke^{-2x}\,dx = 1$$

$$\left[-\frac{k}{2}e^{-2x}\right]_0^\infty = 1$$

$$0 - \left(-\frac{k}{2}\right) = 1$$

$$\frac{k}{2} = 1$$

$$k = 2$$

Question 13 (b)

Criteria	Marks
• Finds correct exact value expression for the distribution median	2
• Sets up the proper definite probability integral equal to 0.5	1

Sample answer:

$$\int_0^m 2e^{-2x}\,dx = 0.5$$

$$\left[-e^{-2x}\right]_0^m = 0.5$$

$$-e^{-2m} + 1 = 0.5$$

$$e^{-2m} = 0.5$$

$$-2m = \ln(0.5)$$

$$m = \frac{\ln 2}{2}$$

Question 13 (c)

Criteria	Marks
• Provides the correct conditional value evaluated to three decimals	2
• Uses conditional probability equation	1

Sample answer:

$$P(X \le 1 \mid X \ge 0.5) = \frac{P(0.5 \le X \le 1)}{P(X \ge 0.5)}$$

$$P(X \ge 0.5) = \int_{0.5}^{\infty} 2e^{-2x}\,dx$$

$$= e^{-1}$$

$$P(0.5 \le X \le 1) = \int_{0.5}^{1} 2e^{-2x}\,dx$$

$$= e^{-1} - e^{-2}$$

$$\text{Probability} = \frac{e^{-1} - e^{-2}}{e^{-1}}$$

$$= 1 - e^{-1}$$

$$\approx 0.632$$

Question 14 (a)

Criteria	Marks
• Successfully derives the given volume formula expression statement	2
• Expresses height parameters correctly utilizing open cylinder surface dimensions	1

Sample answer:

$$A = \pi r^2 + 2\pi r h$$

$$1200\pi = \pi r^2 + 2\pi r h$$

$$h = \frac{1200 - r^2}{2r}$$

$$V = \pi r^2 h$$

$$= \pi r^2 \left(\frac{1200 - r^2}{2r} \right)$$

$$V = 600\pi r - \frac{1}{2}\pi r^3$$

Question 14 (b)

Criteria	Marks
• Provides all correct dimensions and completes secondary verification paths perfectly	4
• Finds correct solutions for both radius and height measurements	3
• Evaluates first derivatives equal to zero to identify critical root locations	2
• Calculates the accurate derivative mapping function expression form	1

Sample answer:

$$\frac{dV}{dr} = 600\pi - \frac{3}{2}\pi r^2$$

$$0 = 600\pi - \frac{3}{2}\pi r^2$$

$$r^2 = 400$$

$$r = 20 \text{ cm}$$

$$h = \frac{1200 - 20^2}{2(20)}$$

$$= 20 \text{ cm}$$

$$\frac{d^2V}{dr^2} = -3\pi r$$

$$\frac{d^2V}{dr^2} = -60\pi$$

$-60\pi < 0 \implies$ Max Volume

Question 15 (a)

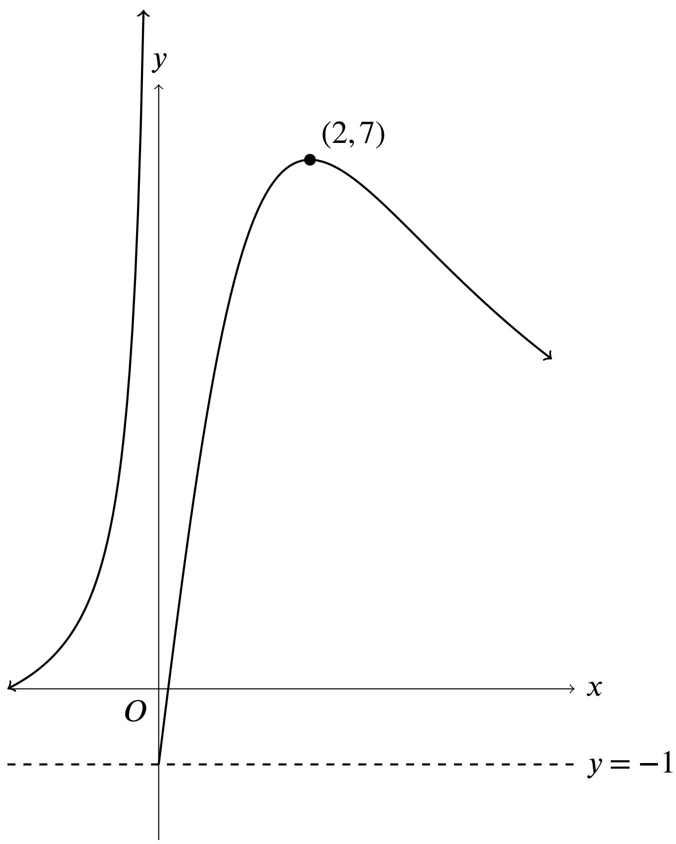
Criteria	Marks
• Correctly sketches the graph with all updated key features labeled explicitly	3
• Labels updated horizontal asymptotes and shifts critical turning boundaries	2
• Incorporates uniform vertical scaling expansions, or equivalent merit	1

Sample answer:

Turning Point: $(2,4) \to (2, 2\times4-1) \implies (2,7)$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 0 \to y = -1$



Question 15 (b)

Criteria	Marks
• Outlines correct intervals for both domain and range definitions	2
• Identifies valid boundary limits for one structural scope definition	1

Sample answer:

Domain: $x \in \mathbb{R}, x \neq 0$

Range: $y > -1$

Question 16 (a)

Criteria	Marks
• Provides the correct volume quantity of bottles required	2
• Equates marginal revenue statements directly against marginal cost paths	1

Sample answer:

$$\frac{dR}{dx} = \frac{dC}{dx}$$

$$150 - 0.4x = 0.2x + 30$$

$$120 = 0.6x$$

$x = 200$ bottles

Question 16 (b)

Criteria	Marks
• Calculates the accurate peak financial profit balance achieved	3
• Formulates constant parameter factors using provided fixed cost structures	2
• Integrates net revenue operational models correctly to produce initial polynomials	1

Sample answer:

$$P(x) = \int \left(\frac{dR}{dx} - \frac{dC}{dx}\right) dx$$

$$= \int (120 - 0.6x)\, dx$$

$$= 120x - 0.3x^2 + K$$

$$-5000 = 120(0) - 0.3(0)^2 + K \implies K = -5000$$

$$P(200) = 120(200) - 0.3(200)^2 - 5000$$

$$= 24000 - 12000 - 5000$$

$$= \$7,000$$

Question 17 (a)

Criteria	Marks
• Successfully demonstrates chronological compounding progression patterns clearly	2
• Charts matching baseline compound expressions over individual sub-intervals	1

Sample answer:

$A_1 = 2000(1.04)$

$A_2 = 2000(1.04) + 2000(1.04)^2$

$A_3 = 2000(1.04) + 2000(1.04)^2 + 2000(1.04)^3$

Question 17 (b)

Criteria	Marks
• Provides the correct final financial balance rounded to nearest dollar unit	4
• Correctly evaluates the parameterized geometric system components	3
• Forms the proper generalized summation formatting expression structure	2
• Recognizes standard geometric properties from sequence structures	1

Sample answer:

$$A_n = 2000(1.04) + \cdots + 2000(1.04)^n$$

$$= \frac{2000(1.04)\left[(1.04)^n - 1\right]}{1.04 - 1}$$

$$A_{25} = \frac{2080\left[(1.04)^{25} - 1\right]}{0.04}$$

$$\approx \$86,631$$

Question 18 (a)

Criteria	Marks
• Successfully derives the targeted derivative expression form	2
• Applies product or chain operations correctly onto composite targets	1

Sample answer:

$$f'(x) = 1 \cdot \sqrt{4-x^2} + x \cdot \left(\frac{1}{2}(4-x^2)^{-\frac{1}{2}} \cdot (-2x)\right)$$

$$= \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}}$$

$$= \frac{(4-x^2)-x^2}{\sqrt{4-x^2}}$$

$$f'(x) = \frac{4-2x^2}{\sqrt{4-x^2}}$$

Question 18 (b)

Criteria	Marks
• Provides the correct exact evaluation constant value	2
• Deconstructs the integrand algebraically into the derivative part and a standard form	1

Sample answer:

$$\frac{x^2-2x-2}{\sqrt{4-x^2}} = -\frac{1}{2}\left(\frac{4-2x^2}{\sqrt{4-x^2}}\right) - \frac{2x}{\sqrt{4-x^2}}$$

$$= -\frac{1}{2}f'(x) - 2x(4-x^2)^{-\frac{1}{2}}$$

$$\int_0^1 \frac{x^2-2x-2}{\sqrt{4-x^2}}\,dx = \left[-\frac{1}{2}x\sqrt{4-x^2} + 2\sqrt{4-x^2}\right]_0^1$$

$$= \left[\frac{4-x}{2}\sqrt{4-x^2}\right]_0^1$$

$$= \left(\frac{3}{2}\sqrt{3}\right) - \left(\frac{4}{2}\sqrt{4}\right)$$

$$= \frac{3\sqrt{3}}{2} - 4$$

Question 19 (a)

Criteria	Marks
• Provides the correct total defective percentage using the empirical rule	2
• Calculates standard deviation distance thresholds (z-scores) for boundaries	1

Sample answer:

$$\text{Lower Boundary Deviation} = \frac{0.370 - 0.400}{0.015}$$

$$= -2\sigma$$

$$\text{Upper Boundary Deviation} = \frac{0.415 - 0.400}{0.015}$$

$$= +1\sigma$$

$$\text{Percentage between } -2\sigma \text{ and } +1\sigma = \frac{0.95}{2} + \frac{0.68}{2}$$

$$= 0.475 + 0.340$$

$$= 81.5\% \quad \text{(non-defective rate)}$$

$$\text{Defective Percentage} = 100\% - 81.5\%$$

$$= 18.5\%$$

Question 19 (b)

Criteria	Marks
• Calculates the accurate updated standard deviation parameter value	2
• Links percentage requirements correctly to standard empirical score positions	1

Sample answer:

$z = 2$ (maps from non-defective rate threshold)

$$2 = \frac{0.425 - 0.400}{\sigma}$$

$$2\sigma = 0.025$$

$$\sigma = 0.0125 \text{ mm}$$

Question 20 (a)

Criteria	Marks
• Determines the complete correct solution ranges for parameter x	3
• Expresses the baseline absolute trigonometric restriction statement properly	2
• Recalls proper limiting sum convergence rules based on standard ratios	1

Sample answer:

$$|r| < 1$$

$$\left|\frac{1}{2}\sec\left(2x-\frac{\pi}{6}\right)\right| < 1$$

$$\left|\cos\left(2x-\frac{\pi}{6}\right)\right| > \frac{1}{2}$$

Let $\theta = 2x - \frac{\pi}{6}$ where $-\frac{\pi}{6} \le \theta \le \frac{11\pi}{6} \implies -\frac{\pi}{3} < \theta < \frac{\pi}{3}$ or $\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

$$-\frac{\pi}{3} < 2x-\frac{\pi}{6} < \frac{\pi}{3} \implies -\frac{\pi}{6} < 2x < \frac{\pi}{2}$$

$$\implies -\frac{\pi}{12} < x < \frac{\pi}{4}$$

$$\frac{2\pi}{3} < 2x-\frac{\pi}{6} < \frac{\pi}{3} \implies \frac{5\pi}{6} < 2x < \frac{3\pi}{2}$$

$$\implies \frac{5\pi}{12} < x < \frac{3\pi}{4}$$

Apply domain restriction $0 \le x \le \pi$:

$$\therefore \quad 0 \le x < \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{12} < x < \frac{3\pi}{4} \quad \text{or} \quad \frac{11\pi}{12} < x \le \pi$$

Question 20 (b)

Criteria	Marks
• Resolves exact angles correctly satisfying interval boundaries	2
• Simplifies operational limiting sum equations down to base cosines	1

Sample answer:

$$\frac{1}{1-\frac{1}{2}\sec\left(2x-\frac{\pi}{6}\right)}=\frac{4}{4-\sqrt{2}}$$

$$1-\frac{1}{2}\sec\left(2x-\frac{\pi}{6}\right)=1-\frac{\sqrt{2}}{4}$$

$$\cos\left(2x-\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}$$

$$2x-\frac{\pi}{6}=\frac{\pi}{4}$$

$$2x=\frac{5\pi}{12}$$

$$x=\frac{5\pi}{24}$$

Question 21 (a)

Criteria	Marks
• Outlines the accurate time delay metric in monthly units	2
• Identifies absolute maximized index milestones for individual components	1

Sample answer:

$$\frac{\pi t_B}{6} = \frac{\pi}{2} \implies t_B = 3$$

$$\frac{\pi t_C}{6} - \frac{\pi}{4} = \frac{\pi}{2} \implies t_C = 4.5$$

Time lag $= t_C - t_B$

$= 4.5 - 3$

$= 1.5$ months

Question 21 (b)

Criteria	Marks
• Provides correct exact numeric evaluation alongside operational context units	3
• Substitutes targeting indices correctly into companion rate equations	2
• Differentiates tracking populations successfully using basic rules	1

Sample answer:

$t = 6$ (inflection point layout milestone)

$$C'(t) = 40\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi t}{6} - \frac{\pi}{4}\right)$$

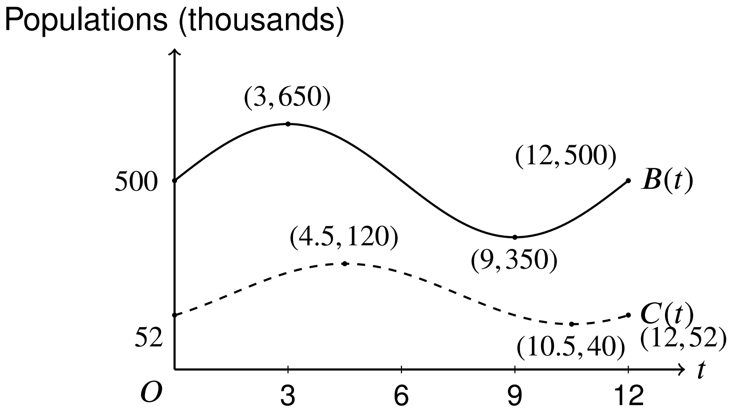
$$C'(6) = \frac{20\pi}{3}\cos\left(\pi - \frac{\pi}{4}\right)$$

$$= -\frac{10\sqrt{2}\pi}{3} \text{ thousand cicadas/month}$$

Question 21 (c)

Criteria	Marks
• Correctly sketches both population tracking paths showing appropriate phase steps	2
• Sketches one valid periodic function path layout across limits	1

Sample answer:



Question 22

Criteria	Marks
• Identifies all correct exact angle roots inside targeted intervals	2
• Expresses the trigonometric identity structural layout into standard quadratic setups	1

Sample answer:

$$\tan 2\theta + \frac{1}{\tan 2\theta} = 4$$

$$\tan^2 2\theta - 4\tan 2\theta + 1 = 0$$

$$\tan 2\theta = \frac{4 \pm \sqrt{12}}{2}$$

$$= 2 \pm \sqrt{3}$$

$$2\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

$$\theta = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}$$

Question 23 (a)

Criteria	Marks
• Provides accurate exact coordinates and verifies stationary natures seamlessly	3
• Isolates critical functional roots from candidate numerator terms	2
• Differentiates given terms correctly via quotient configuration templates	1

Sample answer:

$$f'(x) = \frac{\frac{1}{x}(x^2) - 2x\ln x}{x^4}$$

$$= \frac{1 - 2\ln x}{x^3}$$

$$0 = 1 - 2\ln x$$

$$\ln x = \frac{1}{2}$$

$$x = \sqrt{e}$$

$$y = \frac{\ln\left(\sqrt{e}\right)}{\left(\sqrt{e}\right)^2}$$

$$= \frac{1}{2e}$$

x	1	$\sqrt{e} \approx 1.65$	2
$f'(x)$	1	0	≈ -0.05
Slope	Increasing (/)	Horizontal (–)	Decreasing (\)

$\therefore$ Local Maximum at $\left(\sqrt{e}, \frac{1}{2e}\right)$

Question 23 (b)

Criteria	Marks
• Pinpoints accurate exact analytical coordinates for inflection points	2
• Evaluates secondary derivative equations equal to zero structures	1

Sample answer:

$$f''(x) = \frac{-\frac{2}{x}(x^3) - 3x^2(1 - 2\ln x)}{x^6}$$

$$= \frac{6\ln x - 5}{x^4}$$

$$0 = \frac{6\ln x - 5}{x^4}$$

$$0 = 6\ln x - 5$$

$$\ln x = \frac{5}{6}$$

$$x = e^{\frac{5}{6}}$$

$$y = \frac{5}{6e^{5/3}}$$

Coordinate: $\left(e^{\frac{5}{6}}, \frac{5}{6e^{5/3}}\right)$

Question 24 (a)

Criteria	Marks
• Demonstrates single segment trapezoidal expansion patterns properly	1

Sample answer:

$$A_k = \frac{1}{2}(1)\left[f(k) + f(k+1)\right]$$

$$= \frac{1}{2}\left[\ln\left(1+\frac{1}{k}\right) + \ln\left(1+\frac{1}{k+1}\right)\right]$$

$$= \frac{1}{2}\left[\ln\left(\frac{k+1}{k}\right) + \ln\left(\frac{k+2}{k+1}\right)\right]$$

Question 24 (b)

Criteria	Marks
• Proves total collection structure using telescoping summation expansions	3
• Organizes logarithmic pairs correctly highlighting cancelation tracks	2
• Synthesizes complete structural sigma layout sequences effectively	1

Sample answer:

$$A_{\text{total}} = \sum_{k=1}^{n} A_k$$

$$= \frac{1}{2}\left[\sum_{k=1}^{n}\ln\left(\frac{k+1}{k}\right) + \sum_{k=1}^{n}\ln\left(\frac{k+2}{k+1}\right)\right]$$

$$= \frac{1}{2}\left[\ln(n+1) - \ln 1 + \ln(n+2) - \ln 2\right]$$

$$= \frac{1}{2}\left[\ln(n+1) + \ln\left(\frac{n+2}{2}\right)\right]$$

Question 24 (c)

Criteria	Marks
• Expresses the correct simplified form constant inside logarithm terms	2
• Substitutes designated integer counts accurately into operational formulas	1

Sample answer:

$$A = \frac{1}{2}\left[\ln(63) + \ln\left(\frac{64}{2}\right)\right]$$

$$= \frac{1}{2}[\ln 63 + \ln 32]$$

$$= \frac{1}{2}\ln(2016)$$

$$= \ln(\sqrt{2016})$$

Question 25 (a)

Criteria	Marks
• Points out proper core problem divisions using Pareto thresholds	1

Sample answer:

Classifications tracking directly to standard thresholds:

Weather and Mechanical Delays

Question 25 (b)

Criteria	Marks
• Evaluates metrics accurately and concludes structural consistency statements safely	2
• Extracts appropriate range or median numbers from standard box plots	1

Sample answer:

$\text{IQR}_A = 55 - 20$

$= 35$ minutes

$\text{IQR}_B = 65 - 40$

$= 25$ minutes

Hub B displays a lower IQR, establishing greater consistency in delays.

Question 25 (c)

Criteria	Marks
• Solves for accurate percentage rates relating to the targeted condition	2
• Accounts for single component raw volume attributes correctly from tables	1

Sample answer:

$$\begin{aligned}
\text{Total over 55 mins} &= 0.25 \times 500 \\
&= 125 \\
\text{Mechanical} &= 120 \times 0.25 \\
&= 30 \\
\text{Customs/Route} &= 100 \times 0.05 \\
&= 5 \\
\text{Weather over 55 mins} &= 125 - 30 - 5 \\
&= 90 \\
\text{Percentage Rate} &= \frac{90}{280} \times 100\% \\
&\approx 32.14\%
\end{aligned}$$

Question 26 (a)

Criteria	Marks
• Explains biological meanings perfectly for both slope parameters and correlation indices	2
• Provides appropriate description for one tracking indicator	1

Sample answer:

Gradient (-4.5) : Each $1°$C temperature increase reduces average coverage by 4.5%.

Correlation (-0.87) : Indicates a strong negative linear association between variables.

Question 26 (b)

Criteria	Marks
• Calculates accurate localized residual system metrics	1

Sample answer:

$$\text{Predicted coverage} = -4.5(24) + 142$$

$$= 34\%$$

$$\text{Residual difference} = 35\% - 34\%$$

$$= 1\%$$

Question 27 (a)

Criteria	Marks
• Verifies single baseline dimension segments correctly via trigonometry links	1

Sample answer:

$$\tan\beta = \frac{h}{AM}$$

$$AM = \frac{h}{\tan\beta}$$

$$AM = h\cot\beta$$

Question 27 (b)

Criteria	Marks
• Successfully links 3D spatial dimensions perfectly to prove targeted structures	3
• Introduces Pythagoras configurations accurately matching baseline path maps	2
• Represents horizontal length segments properly via compound variables	1

Sample answer:

$$OM = h\cot\alpha$$

$$d + h\cot\beta = h\cot\alpha$$

$$d = h(\cot\alpha - \cot\beta)$$

$$BM^2 = AB^2 + AM^2$$

$$(h\cot\gamma)^2 = d^2 + (h\cot\beta)^2$$

$$h^2\cot^2\gamma = h^2(\cot\alpha - \cot\beta)^2 + h^2\cot^2\beta$$

$$\cot^2\gamma = \cot^2\beta + (\cot\alpha - \cot\beta)^2$$

Question 28

Criteria	Marks
• Provides the correct exact evaluation for constant parameter a	3
• Formulates secondary matching models based on expectation conditions	2
• Applies regular normalization unity rules across total distributions	1

Sample answer:

$$\sum P(x) = 1$$

$$\frac{5.5a}{E(X)} = 1$$

$$E(X) = 5.5a \quad \text{— (Equation 1)}$$

$$E(X) = \sum x \cdot P(x)$$

$$= 1\left(\frac{a}{E(X)}\right) + 2\left(\frac{a}{E(X)}\right) + 3\left(\frac{a}{E(X)}\right) + 4\left(\frac{2a}{E(X)}\right) + 5\left(\frac{a}{2E(X)}\right)$$

$$E(X) = \frac{a + 2a + 3a + 8a + 2.5a}{E(X)}$$

$$E(X) = \frac{16.5a}{E(X)}$$

$$E(X)^2 = 16.5a \quad \text{— (Equation 2)}$$

Substitute Equation 1 into Equation 2:

$$(5.5a)^2 = 16.5a$$

$$30.25a^2 = 16.5a$$

$$30.25a = 16.5 \quad \text{(since } a > 0\text{)}$$

$$a = \frac{16.5}{30.25}$$

$$a = \frac{6}{11}$$

Question 29 (a)

Criteria	Marks
• Establishes targeted simplified integral statements correctly via transformations	2
• Introduces symmetric functional operations appropriately onto limits	1

Sample answer:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x}\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(-x)^2 \cos(-x)}{1+e^{-x}}\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^x x^2 \cos x}{1+e^x}\,dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+e^x) x^2 \cos x}{1+e^x}\,dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\,dx$$

$$I = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \cos x\,dx$$

Question 29 (b)

Criteria	Marks
• Justifies bounding inequality positions perfectly via standard function comparisons	3
• Integrates upper bounding tracking shapes correctly over boundaries	2
• Recognizes trigonometric coordinate boundaries across domains	1

Sample answer:

$\cos x \leq 1$

$$I < \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2(1)\,dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(\frac{\pi^3}{24} - \left(-\frac{\pi^3}{24}\right)\right)$$

$$= \frac{\pi^3}{24}$$

Since $\cos x < 1$ everywhere else on domain, $I < \dfrac{\pi^3}{24}$.